

Higher derivative corrections to DBI action at α'^2 order

Komeil Babaei Velni^{1*} and Ali Jalali^{2†}

¹Department of Physics, University of Guilan,
P.O. Box 41335-1914, Rasht, Iran

²Department of Physics, Ferdowsi University of Mashhad,
P.O. Box 1436, Mashhad, Iran

Abstract

We use the compatibility of D-brane action with linear off-shell T-duality and linear on-shell S-duality as guiding principles to find all world volume couplings of one massless closed and three massless open strings at order α'^2 in type II superstring theories in flat space-time.

Keywords: T-duality; S-duality; Higher-derivative Couplings

*babaeivelni@guilan.ac.ir

†ali.jalali@stumail.um.ac.ir

1 Introduction and Results

D-branes are non-perturbative objects in superstring theory which play a central role in exploring different aspects of the theory; from statistical computation of black hole entropy [1] to realization of the AdS/CFT correspondence [2] or appearance of noncommutative geometry in string theory [3]. However, these objects can be studied by using a perturbative method thanks to their description in terms of open strings with Dirichlet boundary conditions [4].

Much of the importance of D_p -branes stems from the fact that they provide a remarkable way of introducing nonabelian gauge symmetries in string theory. Nonabelian gauge fields naturally appear confined to the world volume of multiple coincident D_p -branes [5, 6].

The low energy effective field theory of D_p -branes in type II superstring theories consists of the Dirac-Born-Infeld (DBI) [7] and the Chern-Simons (CS) actions [8], *i.e.*,

$$S_p = -T_p \int d^{p+1}x e^{-\phi} \sqrt{-\det(P[g+B]_{ab} + F_{ab})} + T_p \int e^F P[e^B C] \quad (1)$$

where $P[\dots]$ is the pull-back operator which projects the spacetime tensors to the world volume, *e.g.*, $P[g]_{ab} = \partial X^\mu / \partial \sigma^a \partial X^\nu / \partial \sigma^b g_{\mu\nu} = \tilde{G}_{ab}$. The dependence of the closed string fields on the transverse coordinates appears in the action via the Taylor expansion [9]. In the literature, there is a factor of $2\pi\alpha'$ in front of gauge field strength F_{ab} while here, we normalize the gauge field to absorb this factor.

The curvature, the second fundamental form and the dilaton corrections to the DBI action at order α'^2 in the string frame are reported in [10, 11, 12] as*

$$S_p^{DBI} \supset -\frac{\pi^2 \alpha'^2 T_p}{48} \int d^{p+1}x e^{-\Phi} \sqrt{-\tilde{G}} \left[(R_T)_{abcd} (R_T)^{abcd} - 2(\mathcal{R}_T)_{ab} (\mathcal{R}_T)^{ab} - (R_N)_{abij} (R_N)^{abij} + 2\bar{\mathcal{R}}_{ij} \bar{\mathcal{R}}^{ij} \right] \quad (2)$$

where the curvatures $(R_T)_{abcd}$ and $(R_N)^{abij}$ are related to the projections of the bulk Riemann curvatures into world volume and transverse spaces via the Gauss-Codazzi equations, *i.e.*,

$$\begin{aligned} (R_T)_{abcd} &= R_{abcd} + \delta_{ij} (\Omega_{ac}^i \Omega_{bd}^j - \Omega_{ad}^i \Omega_{bc}^j) \\ (R_N)_{ab}^{ij} &= R_{ab}^{ij} + g^{cd} (\Omega_{ac}^i \Omega_{bd}^j - \Omega_{ac}^j \Omega_{bd}^i) \end{aligned} \quad (3)$$

In addition, the curvatures $(\mathcal{R}_T)_{ab}$ and $\bar{\mathcal{R}}_{ij}$ are related to the Riemann curvatures, the second fundamental form and the dilaton via the following relations:

$$\begin{aligned} (\mathcal{R}_T)_{ab} &= g^{cd} (R_T)_{cadb} + \partial_a \partial_b \Phi \\ (\bar{\mathcal{R}})^{ij} &= g^{ab} R_a^i b^j + g^{ab} g^{cd} (\Omega_{ac}^i \Omega_{bd}^j - \Omega_{ab}^i \Omega_{cd}^j) + \partial_i \partial_j \Phi \end{aligned} \quad (4)$$

*Our index convention is that the Greek letters (μ, ν, \dots) are the indices of the space-time coordinates, the Latin letters (a, d, c, \dots) are the world-volume indices and the letters (i, j, k, \dots) are the normal bundle indices. The Killing index in the reduction of 10-dimensional spacetime to 9-dimensional spacetime is y .

in which the world volume indices are raised by the inverse of the pull-back metric. In static gauge, the second fundamental form includes the second derivative of the transverse scalar fields, *i.e.*, $\Omega_{ab}^i = \partial_a \partial_b \phi^i - \tilde{\Gamma}_{ab}^c \partial_c \phi^i + \Gamma_{ab}^i$. Thus, the action (2) includes the couplings of one graviton or dilaton and two scalar fields. All other couplings between one NSNS and two NS fields at order α'^2 have been found in [13] by requiring (2) to be invariant under linear T-duality and also the couplings to be consistent with the corresponding S-matrix elements. The couplings in the string frame are [13]

$$S_p^{DBI} \supset -\frac{\pi^2 \alpha'^2 T_p}{12} \int d^{p+1}x e^{-\Phi} \sqrt{-\tilde{G}} \left[\mathcal{R}_{bd} (\partial_a F^{ab} \partial_c F^{cd} - \partial_a F_c^d \partial^c F^{ab}) + \frac{1}{2} R_{bdce} \partial^c F^{ab} \partial^e F_a^d \right. \\ \left. + \frac{1}{4} \mathcal{R}_d^d (\partial_a F^{ab} \partial_c F_b^c + \partial_b F_a^c \partial_c F^{ab}) + \Omega_a^{ai} \partial_d H_c^d \partial_i F^{bc} \right. \\ \left. - \Omega^{bai} \left(\partial_b F_a^c \partial_d H_c^d + \partial^d F_a^c \partial_i H_{bcd} - \frac{1}{2} \partial^d F_a^c \partial_c H_{bdi} \right) \right] \quad (5)$$

where \mathcal{R}_a^a is given by

$$\mathcal{R}_a^a = R^a{}_{ab} + 2\partial^a \partial_a \Phi \quad (6)$$

which is invariant under linear T-duality. In (5), the Riemann curvatures and the field strength $H = dB$, are the pull-back of the spacetime curvature and B-field onto world volume and transverse spaces respectively. For example

$$R_{abcd} = \partial_a X^\alpha \partial_b X^\beta \partial_c X^\mu \partial_d X^\nu R_{\alpha\beta\mu\nu}$$

or

$$\partial_a H_{bci} \Omega_{de}^i = \perp^{\alpha\beta} \partial_a X^\mu \partial_b X^\nu \partial_c X^\rho \partial_d X^\sigma \partial_e X^\zeta \partial_\mu H_{\nu\rho\alpha} \Omega_{\sigma\zeta\beta},$$

in which $\partial_a X^\mu$ is the pull-back operator into the world volume while $\perp^{\alpha\beta}$ is the pull-back operator into the transverse space [14] *i.e.*,

$$\perp^{\alpha\beta} = G^{\alpha\beta} - \tilde{G}^{\alpha\beta}, \quad \tilde{G}^{\alpha\beta} = \frac{\partial X^\alpha}{\partial \sigma^a} \frac{\partial X^\beta}{\partial \sigma^b} \tilde{G}^{ab}.$$

In the last equation, $G^{\alpha\beta}$ is the first fundamental form and \tilde{G}^{ab} is the inverse of the pull-back metric. In the static gauge, *i.e.*, $X^a = \sigma^a$ and $X^i = \phi^i$, the components of the projection operator $\perp^{\alpha\beta}$ become $\perp^{ab} = 0$, $\perp^{ai} = -\partial^a \phi^i$ and $\perp^{ij} = \eta^{ij}$ to the linear order of transverse scalar field in which we are interested. So one can extend the couplings (5) from one NSNS closed string and two open string couplings to the one NSNS and three open NS string. These couplings are called as $S_{p,l}$.

On the other hand the four open string corrections to the DBI action at α'^2 order have been found in [15]. These couplings, that appear in three families, include the couplings of four scalar fields, four gauge fields and two scalars and two gauge fields. The four scalar field

couplings are [15]:

$$S_{\phi\phi\phi\phi} = -\frac{\pi^2\alpha'^2 T_p}{48} \int d^{p+1}x \sqrt{-\tilde{G}} \left[4\Omega^{abi}\Omega_{ab}{}^j\Omega^{cd}{}_i\Omega_{cdj} - 4\Omega^{abi}\Omega_a{}^c{}_i\Omega_b{}^{dj}\Omega_{cdj} \right. \\ \left. + 4\Omega_a{}^i\Omega^{bc}{}_i\Omega_b{}^{dj}\Omega_{cdj} - 6\Omega_a{}^i\Omega_b{}^j\Omega^{cd}{}_i\Omega_{cdj} + 2\Omega_a{}^i\Omega_b{}^j\Omega_c{}^k\Omega^d{}_{dj} \right] \quad (7)$$

In the above effective action, at the order of four fields, in addition to the four scalar couplings there are couplings between one graviton and three scalar fields, two gravitons and two scalar fields, three gravitons and one scalar field and four gravitons. In this paper we are interested in the couplings between one graviton and three scalars of Eq. (7). As discussed in [15], the consistency of couplings in (7) with the T-duality and S-duality can be used to determine the four gauge field couplings as follow:

$$S_{ffff} = -\frac{\pi^2\alpha'^2 T_p}{48} \int d^{p+1}x \sqrt{\tilde{G}} \left[2\partial^a \mathcal{F}^{bc} \partial_b \mathcal{F}_a{}^d \partial_c \mathcal{F}^{ef} \partial_d \mathcal{F}_{ef} \right. \\ - \frac{1}{2} \partial_a \mathcal{F}_{bc} \partial^a \mathcal{F}^{bc} \partial_d \mathcal{F}_{ef} \partial^d \mathcal{F}^{ef} - 2\partial^a \mathcal{F}_a{}^b \partial^c \mathcal{F}_{bc} \partial_d \mathcal{F}_{ef} \partial^d \mathcal{F}^{ef} - 4\partial^a \mathcal{F}^{bc} \partial_b \mathcal{F}_c{}^d \partial_d \mathcal{F}^{ef} \partial_e \mathcal{F}_{af} \\ \left. + 2\partial_a \mathcal{F}^{de} \partial^a \mathcal{F}^{bc} \partial_f \mathcal{F}_{ce} \partial^f \mathcal{F}_{bd} + 2\partial^a \mathcal{F}_a{}^b \partial^c \mathcal{F}_b{}^d \partial_d \mathcal{F}_c{}^e \partial^e \mathcal{F}_{ef} + 6\partial^a \mathcal{F}_a{}^b \partial_c \mathcal{F}^{ef} \partial^c \mathcal{F}_b{}^d \partial_d \mathcal{F}_{ef} \right] \quad (8)$$

in which $\mathcal{F}_{ab} = F_{ab} + B_{ab}$ is the gauge invariant combination of B-field and gauge field. It should be pointed out that due to the definition of \mathcal{F}_{ab} , the couplings in Eq. (8) also include two B-field and two gauge field couplings, three B-field and one gauge field couplings and four B-field couplings that we do not consider in this work. Moreover, the couplings of two gauge fields and two scalar fields appear in the following effective action [15]:

$$S_{\phi\phi ff} = -\frac{\pi^2\alpha'^2 T_p}{48} \int d^{p+1}x \sqrt{-\tilde{G}} \left[4\Omega_{cd}{}^i\Omega^e{}_{ei}\partial_a \mathcal{F}_b{}^d \partial^a \mathcal{F}^{bc} - 8\Omega_c{}^{ei}\Omega_{dei}\partial_a \mathcal{F}_b{}^d \partial^a \mathcal{F}^{bc} \right. \\ + 12\Omega_c{}^{ei}\Omega_{dei}\partial^a \mathcal{F}^{bc} \partial_b \mathcal{F}_a{}^d - 4\Omega_{cd}{}^i\Omega^e{}_{ei}\partial^a \mathcal{F}^{bc} \partial_b \mathcal{F}_a{}^d - 4\Omega_d{}^i\Omega^e{}_{ei}\partial^a \mathcal{F}_a{}^b \partial^c \mathcal{F}_{bc} \\ \left. - 4\Omega_c{}^{ei}\Omega_{dei}\partial^a \mathcal{F}_a{}^b \partial^c \mathcal{F}_b{}^d + 12\Omega_{cd}{}^i\Omega^e{}_{ei}\partial^a \mathcal{F}_a{}^b \partial^c \mathcal{F}_b{}^d + 8\Omega_{ac}{}^i\Omega_{dei}\partial^a \mathcal{F}^{bc} \partial^d \mathcal{F}_b{}^e \right] \quad (9)$$

Through the \mathcal{F}_{ab} definition the couplings in (8) include either one graviton, one scalar and two gauge fields or one B-field, one gauge field and two scalar field couplings. The couplings in (9), in addition to the mentioned ones, include the couplings of two closed strings and two open strings, three closed strings and one open string and four closed strings that we do not consider here.

In this paper, we use the compatibility of couplings in (5), (7), (8) and (9) and all other couplings of one NSNS closed string and three NS open strings with off-shell linear T-duality and on-shell linear S-duality to find all couplings of one NSNS and three open string couplings at α'^2 order. We write all contractions of one NSNS and three NS fields at order α'^2 with unknown coefficients. By imposing consistency of the couplings with linear T-duality and

linear S-duality, one can fix the coefficients. We find the graviton couplings as:

$$\begin{aligned}
S_{h\phi\phi} = & -\frac{\pi^2\alpha'^2 T_p}{48} \int d^{p+1}x \sqrt{-\tilde{G}} F_{ab} \left[R_d{}^e{}_{ei} \Omega_a{}^c{}^i \partial_b F_c{}^d - 2R_d{}^e{}_{ei} \Omega_c{}^{ci} \partial_b F_a{}^d \right. \\
& - R_{cide} \Omega_a{}^i \partial_b F^{de} + R_d{}^e{}_{ei} \Omega^{dci} \partial_c F_{ab} + R_{bedi} \Omega^{dci} \partial_c F_a{}^e - 3R_{bide} \Omega^{dci} \partial_c F_a{}^e \\
& - 2R_d{}^e{}_{ei} \Omega_a{}^c{}^i \partial_c F_b{}^d + 2R_c{}^e{}_{ei} \Omega_a{}^c{}^i \partial_d F_b{}^d - R_{aebi} \Omega^{dci} \partial_d F_c{}^e + R_{aebi} \Omega_c{}^{ci} \partial_d F^{de} \\
& + R_{bice} \Omega_a{}^i \partial_d F^{de} - 2R_{bcdi} \Omega^{dci} \partial_e F_a{}^e - R_{adbi} \Omega^{dci} \partial_e F_c{}^e - R_{cedi} \Omega^{dci} \partial^e F_{ab} \\
& + 3R_{bedi} \Omega^{dci} \partial^e F_{ac} - R_{bide} \Omega^{dci} \partial^e F_{ac} - 2R_{bedi} \Omega_c{}^{ci} \partial^e F_a{}^d + R_{bide} \Omega_c{}^{ci} \partial^e F_a{}^d \\
& \left. - 2R_{cedi} \Omega_a{}^i \partial^e F_b{}^d + R_{bedi} \Omega_a{}^i \partial^e F_c{}^d - 2R_{bide} \Omega_a{}^i \partial^e F_c{}^d \right] \quad (10)
\end{aligned}$$

while one B-field, one gauge field and two scalar field couplings could be derived as follows:

$$\begin{aligned}
S_{Bf\phi\phi} = & -\frac{\pi^2\alpha'^2 T_p}{48} \frac{1}{2} \int d^{p+1}x \sqrt{-\tilde{G}} \left[H_{cij} \Omega^{bai} \Omega_a{}^j \partial_d F_b{}^d + \frac{1}{2} H_{dij} \Omega^{bai} \Omega_a{}^j \partial^d F_{bc} \right. \\
& - H_{dij} \Omega^{bai} \Omega^{dcj} \partial_b F_{ac} - 3H_{cij} \Omega_a{}^{ai} \Omega^{cbj} \partial_d F_b{}^d + 3H_{cde} \Omega_{bai} \Omega^{bai} \partial^e F^{cd} \\
& + 5F^{ab} \Omega_a{}^c{}^i \Omega_d{}^{dj} \partial_b H_{cij} + F^{ab} \Omega_a{}^c{}^i \Omega_d{}^j \partial_b H_{dij} - 5F^{ab} \Omega_a{}^c{}^i \Omega_d{}^{dj} \partial_c H_{bij} \\
& - 3F^{ab} \Omega_a{}^c{}^i \Omega_d{}^j \partial_d H_{bij} - 5H_{cde} \Omega_a{}^{ai} \Omega^{cb} \partial_b F^{de} - 5H_{dij} \Omega^{bai} \Omega_a{}^j \partial_c F_b{}^d \\
& + 3F^{ab} \Omega_a{}^c{}^i \Omega^{ed} \partial_e H_{bcd} - F^{ab} \Omega_a{}^c{}^i \Omega_d{}^d \partial_e H_{bc}{}^e + F^{ab} \Omega_a{}^c{}^i \Omega^{ci} \partial_e H_{bd}{}^e \\
& + 5F^{ab} \Omega_a{}^c{}^i \Omega_d{}^{dj} \partial_i H_{bcj} + F^{ab} \Omega_a{}^c{}^i \Omega_d{}^j \partial_i H_{bdj} + F^{ab} \Omega_c{}^{ci} \Omega_d{}^{dj} \partial_j H_{abi} \\
& - F^{ab} \Omega_{dc}{}^j \Omega^{dci} \partial_j H_{abi} - 5F^{ab} \Omega_a{}^c{}^i \Omega_d{}^{dj} \partial_j H_{bci} - F^{ab} \Omega_a{}^c{}^i \Omega_d{}^j \partial_j H_{bdi} \\
& \left. + \frac{1}{2} F^{ab} \Omega_c{}^{ci} \Omega_d{}^d \partial_e H_{ab}{}^e - \frac{1}{2} F^{ab} \Omega_{dci} \Omega^{dci} \partial_e H_{ab}{}^e - \frac{1}{2} H_{bde} \Omega^{bai} \Omega^{dc} \partial_c F_a{}^e \right] \quad (11)
\end{aligned}$$

In addition, one B-field, one gauge field and two scalar field couplings are:

$$\begin{aligned}
S_{Bf\phi\phi} = & -\frac{\pi^2\alpha'^2 T_p}{48} \frac{1}{2} \int d^{p+1}x \sqrt{-\tilde{G}} \left[F^{ab} \partial_d F_c{}^e \partial^d F_a{}^c \partial_f H_{be}{}^f - F^{ab} \partial^d F_a{}^c \partial^e F_{cd} \partial_f H_{be}{}^f \right. \\
& - F^{ab} \partial^d F_a{}^c \partial_e H_{bdf} \partial^f F_c{}^e - H_{bdf} \partial^c F^{ab} \partial^e F_a{}^d \partial^f F_{ce} + F^{ab} \partial^d F_a{}^c \partial_f H_{bce} \partial^f F_d{}^e \quad (12) \\
& + \frac{3}{4} H_{bef} \partial^c F^{ab} \partial_d F^{ef} \partial^d F_{ac} - \frac{5}{4} H_{def} \partial_b F^{ef} \partial_c F_a{}^d \partial^c F^{ab} + \frac{5}{12} H_{cdf} \partial^c F^{ab} \partial^e F_a{}^d \partial^f F_{be} \\
& - \frac{1}{4} H_{def} \partial_a F^{ab} \partial_c F^{ef} \partial^d F_b{}^c - \frac{1}{4} H_{cef} \partial_a F^{ab} \partial_d F^{ef} \partial^d F_b{}^c - \frac{1}{4} H_{cdf} \partial_a F^{ab} \partial_b F^{cd} \partial_e F^{ef} \\
& - \frac{1}{4} F^{ab} \partial_c H_{bef} \partial_d F^{ef} \partial^d F_a{}^c - \frac{1}{4} F^{ab} \partial_c F^{ef} \partial_d H_{bef} \partial^d F_a{}^c - F^{ab} \partial_c F_a{}^c \partial_d F^{de} \partial_f H_{be}{}^f \\
& \left. - \frac{1}{2} F^{ab} \partial^e F^{cd} \partial_f H_{abd} \partial^f F_{ce} - \frac{1}{2} F^{ab} \partial^d F_a{}^c \partial_e H_{bcf} \partial^f F_d{}^e - \frac{1}{2} F^{ab} \partial_c F^{cd} \partial_f H_{abe} \partial^f F_d{}^e \right]
\end{aligned}$$

An outline of the paper is as follows: In the next section, we review the constraints that linear T-duality and S-duality may impose on an effective world volume action. In section 3, we construct all couplings of one NSNS and three NS strings with arbitrary coefficients, and find the coefficients by requiring the consistency of the couplings with the linear dualities.

2 T-duality and S-duality constraints

The S-duality and T-duality transformations on massless fields are generally nonlinear. By constraining the effective actions to be invariant under these nonlinear transformations, one may fix all couplings of bosonic fields [16]. Although this would be a difficult task; see for instance [17, 18, 19] for the case of nonlinear T-duality. In this paper, however, we are interested only in the world volume couplings of one massless closed and three massless open strings states at order α'^2 . We will see in the following that the T-duality transformations on gauge fields and scalar fields are linear while closed string under T-duality do not transform to the open string. By applying this fact, one finds out that the higher derivative couplings of one closed and three open string states have to be invariant under linear T-duality and S-duality transformations.

The full set of nonlinear T-duality transformations has been reported in [26, 27, 28, 29, 30]. We assume a background including a constant dilaton ϕ_0 and a metric that is flat in all directions except for the killing direction y , *i.e.*, y direction is a circle with radius ρ . In addition, we consider quantum fields to be small perturbations around the background, *i.e.*, $G_{\mu\nu} = \eta_{\mu\nu} + 2h_{\mu\nu}$ and $G_{yy} = \rho^2/\alpha'(1 + 2h_{yy})$ where $\mu, \nu \neq y$. For this background, the T-duality transformations are $e^{2\phi_0} = \alpha'e^{2\phi_0}/\rho^2$, $\tilde{G}_{\mu\nu} = \eta_{\mu\nu}$ and $\tilde{G}_{yy} = \alpha'/\rho^2$ while in linear order the quantum fluctuations are as follows:

$$\tilde{\phi} = \phi - \frac{1}{2}h_{yy}, \tilde{h}_{yy} = -h_{yy}, \tilde{h}_{\mu y} = B_{\mu y}, \tilde{B}_{\mu y} = h_{\mu y}, \tilde{h}_{\mu\nu} = h_{\mu\nu}, \tilde{B}_{\mu\nu} = B_{\mu\nu} \quad (13)$$

Also, supposing the χ_y as the transverse scalar, the T-duality transformation of the world volume gauge field, while it is along the Killing direction, is equal to $\tilde{A}_y = \chi_y$. The same is true for $\tilde{\chi}_y = A_y$. Furthermore, the gauge field and the transverse scalar field are invariant under the T-duality when they are not along the Killing direction. This study concerns with applying the above linear T-duality transformations to the quantum fluctuations while imposing the full nonlinear T-duality on the background. The latter needs the DBI effective action to possess an overall factor of $e^{-\Phi}\sqrt{-\tilde{G}}$.

As it is explained in [12, 13, 31], the effective couplings, which are invariant under the mentioned linear T-duality, can be found as follows: First, in the static gauge, we will introduce, in terms of the world volume indices a, b, \dots and the transverse indices i, j, \dots , all couplings on the world volume of D_p -brane which consist of one massless closed and two open string states. The couplings include all contractions of one massless closed and two open string states that will be introduced in the next section and pull-back of couplings in Eq. 5 on the world volume and transverse space. This action will be called S_p , which then will be reduced to an action in 9-dimensional space. This process generates two different actions. In one of these actions, the Killing direction y is a world volume direction, *i.e.*, $a = (\tilde{a}, y)$, which we name as S_p^w , while in the other one the Killing direction y is a transverse direction, $i = (\tilde{i}, y)$ named as S_p^t . Up to some total derivative terms, the S_p^w transformation under the

linear T-duality Eq. (13), named as S_{p-1}^{wT} , should be equal to S_{p-1}^t , *i.e.*,

$$S_{p-1}^{wT} - S_{p-1}^t = 0 \quad (14)$$

The unknown coefficients in the original action S_p will be constrained by this requirement.

On the other hand, the invariance of type IIB theory under S-duality transformations produces another set of constraints on the coefficients of S_p . Due to S-duality, the graviton in Einstein frame, *i.e.*, $G_{\mu\nu}^E = e^{-\Phi/2} G_{\mu\nu}$, and the transverse scalar fields are invariant. Also, the following fields will transform as doublets [32, 33, 34]:

$$\begin{aligned} \mathcal{B} &\equiv \begin{pmatrix} B \\ C^{(2)} \end{pmatrix} \rightarrow (\Lambda^{-1})^T \begin{pmatrix} B \\ C^{(2)} \end{pmatrix} \\ \mathcal{F} &\equiv \begin{pmatrix} *F \\ G(F) \end{pmatrix} \rightarrow (\Lambda^{-1})^T \begin{pmatrix} *F \\ G(F) \end{pmatrix} \end{aligned} \quad (15)$$

where the matrix $\Lambda \in SL(2, Z)$ while $G(F)$ is a nonlinear function of F , Φ , C . Also in the last equation we have $(*F)_{ab} = \epsilon_{abcd} F^{cd}/2$. As reported by [32], to the linear order of the quantum fluctuations and nonlinear background, which we call linear S-duality, one has $G(F) = e^{-\phi_0} F$ where ϕ_0 is the constant dilaton background. The transformation of the dilaton and the RR scalar, where the latter will be called C , appears in $SL(2, Z)$ transformation of the matrix \mathcal{M}

$$\mathcal{M} = e^\Phi \begin{pmatrix} |\tau|^2 C \\ C & 1 \end{pmatrix} \quad (16)$$

in which $\tau = C + ie^{-\Phi}$. This matrix transformation is as follows [32]:

$$\mathcal{M} \rightarrow \Lambda \mathcal{M} \Lambda^T \quad (17)$$

To the zero order of quantum fluctuations and nonlinear order of the background field ϕ_0 , the matrix \mathcal{M} has been found as

$$\mathcal{M}_0 = \begin{pmatrix} e^{-\phi_0} & 0 \\ 0 & e^{\phi_0} \end{pmatrix}, \quad (18)$$

while in the first order we have

$$\delta \mathcal{M} = \begin{pmatrix} -e^{-\phi_0} \phi & e^{\phi_0} C \\ e^{\phi_0} C & e^{\phi_0} \phi \end{pmatrix}. \quad (19)$$

The behavior of the above two matrices under the $SL(2, Z)$ transformation is as (17).

Using the above transformations, one can easily show that there won't be any couplings between one dilaton and two transverse scalars in the Einstein frame. It could be found from $\text{Tr}(\mathcal{M}_0^{-1} \delta \mathcal{M}) = 0$ that one can not be able to make a $SL(2, Z)$ invariant combination of \mathcal{M}_0

and $\delta\mathcal{M}$. This requirement generates a set of constraints on the coefficients of the effective action S_p .

One can easily find the B-fields and three gauge field couplings, appearing in the S-dual multiplet, as follow:

$$\left(\partial(*\mathcal{F}^T)\mathcal{M}_0\partial^2\mathcal{F}\right)\partial\left((*\mathcal{F}^T)\mathcal{M}_0\partial^2\mathcal{B}\right) = e^{-2\phi_0}\left(\partial(*F)\partial(*F) + \partial F\partial F\right)\left(\partial F\partial^2 B + \dots\right) \quad (20)$$

In the above equation the dots refers to the $C^{(2)}$ RR antisymmetric two form couplings which we do not consider in this work. Furthermore, it is possible to show the invariance of the following structures under the linear S-duality transformation.

$$\begin{aligned} R\Omega\mathcal{F}^T\mathcal{M}_0\partial\mathcal{F} &= e^{-\phi_0}R\Omega(*F\partial(*F) + F\partial F) \\ \Omega\partial\mathcal{F}^T\partial\mathcal{M}\partial\mathcal{F} &= \Omega\left(e^{-\phi_0}\partial\Phi\partial F\partial F - e^{-\phi_0}\partial\Phi\partial(*F)\partial(*F) + \dots\right) \end{aligned} \quad (21)$$

Then the couplings of one closed and three open string states on the world volume of D₃-brane should appear, up to total derivative terms, in the above structures. These structures constrain some of the coefficients of the couplings in the effective action S_p .

Following the discussion in [19], we know that for the probe branes action to be constructed, we need to impose the bulk equations of motion at order α'^0 into S_p . Because the world volume couplings which have linear closed string fields are our main interest, the supergravity equations of motion at linear order should be imposed, *i.e.*,

$$\begin{aligned} R + 4\nabla^2\Phi &= 0 \\ R_{\mu\nu} + 2\nabla_{\mu\nu}\Phi &= 0 \\ \nabla^\rho H_{\rho\mu\nu} &= 0 \end{aligned} \quad (22)$$

in which μ, ν, ρ show the bulk indices. These indices could be rewritten based on world volume and transverse indices as follows

$$\begin{aligned} R_{\mu}{}^i{}_{\nu i} &= -2\nabla_{\mu\nu}\Phi - R_{\mu}{}^c{}_{\nu c} \\ \nabla^i{}_i\Phi &= -\nabla^a{}_a\Phi \\ \nabla^i H_{i\mu\nu} &= -\nabla^a H_{a\mu\nu} \end{aligned} \quad (23)$$

This illustrate that the terms on the lhs are not independent. Therefore, the coefficients of the couplings in the effective action S_p , which involve the terms on the left-hand side of the above equation, have to be zero.

3 All contractions

By using “xAct” [35], a mathematica package, all couplings of one massless NSNS state and three massless NS state with unknown coefficients will be derive in this section. Then,

we constrain the coefficients by imposing the consistency of the couplings with both linear T-duality and S-duality.

The couplings of one graviton, two gauge fields and one scalar field have two structures *i.e.*, $R\partial F\Omega F$ and $\Omega\Omega\partial F\partial F$. Each structure has two contractions; the first one is the following:

$$S_{h\phi ff}^{(1)} = -\frac{\pi^2\alpha'^2 T_p}{48} \int d^{p+1}x e^{-\Phi} \sqrt{-\tilde{G}} \left(\eta_1 R_{abci} \partial_d F^{bc} \Omega_e^{ei} F^{ad} + \eta_2 R_{bide} \partial_a F^{de} \Omega_c^{ci} F^{ab} \right. \\ \left. + \eta_3 R_d^e{}_{ei} \partial_b F_a^d \Omega_c^{ci} F^{ab} + \dots + \eta_{58} R_{bide} \partial^e F_c^d \Omega_a^{ci} F^{ab} \right) \quad (24)$$

where η_i are 58 arbitrary coefficients that should be determined by imposing appropriate constraints. By imposing the Bianchi identities and also ignoring total derivative terms, one realizes that not all the mentioned coefficients are independent. One may first find independent coefficients and then impose the constraints. Also, it is possible to first impose the constraints and then ignore the terms that are related by the Bianchi identities and total derivatives. We use the latter approach which is easier to work with computer. Graviton also may appear in the second fundamental form; thus, the couplings of one graviton, one scalar field and two gauge fields appears in the $\Omega\Omega\partial F\partial F$ structure and the second contraction will be found as follow:

$$S_{h\phi ff}^{(2)} = -\frac{\pi^2\alpha'^2 T_p}{48} \int d^{p+1}x e^{-\Phi} \sqrt{-\tilde{G}} \left(\phi_1 \Omega^{bai} \Omega_c^{ci} \partial_b \mathcal{F}^{de} \partial_c \mathcal{F}_{de} + \phi_2 \Omega_a^{ai} \Omega^{cb}{}_i \partial_b \mathcal{F}^{de} \partial_c \mathcal{F}_{de} \right. \\ \left. + \phi_3 \Omega^{bai} \Omega^{dc}{}_i \partial_b \mathcal{F}_{de} \partial_c \mathcal{F}_a^e + \dots + \lambda_8 \Omega_{abi} \Omega^{abi} \Omega_{cdj} \Omega^{cdj} \right) \quad (25)$$

in which $\mathcal{F}_{ab} = F_{ab} + B_{ab}$ is the gauge invariant combination of B-field and gauge field open string. Through the \mathcal{F}_{ab} definition the couplings in (25) include either one graviton, one scalar and two gauge fields or one B-field, one gauge field and two scalar field couplings. It should be pointed out that η_i and ϕ_i are 66 arbitrary coefficients.

To check the consistency of the couplings in (24) and (25) with T-duality, we need some couplings of dilaton and B-field. The dilaton couplings are as follow:

$$S_{\Phi\phi ff}^{(1)} = -\frac{\pi^2\alpha'^2 T_p}{48} \int d^{p+1}x e^{-\Phi} \sqrt{-\tilde{G}} \left(\zeta_1 \Omega^{ab}{}_i \partial_a F^{cd} \partial_b F_{cd} \partial^i \Phi + \zeta_2 \Omega^{ab}{}_i \partial_c F_a^c \partial_d F_b^d \partial^i \Phi \right. \\ \left. + \zeta_3 \Omega^a{}_{ai} \partial_b F^{bc} \partial_d F_c^d \partial^i \Phi + \dots + \zeta_9 \Omega^a{}_{ai} \partial_d F_{bc} \partial^d F^{bc} \partial^i \Phi \right) \quad (26)$$

$$S_{\Phi\phi ff}^{(2)} = -\frac{\pi^2\alpha'^2 T_p}{48} \int d^{p+1}x e^{-\Phi} \sqrt{-\tilde{G}} \left(\chi_1 F^{ab} \Omega_c^{ci} \partial_d F_b^d \partial_i \partial_a \Phi - \chi_2 F^{ab} \Omega_a^{ci} \partial_d F_c^d \partial_i \partial_b \Phi \right. \\ \left. + \chi_3 F^{ab} \Omega^{cdi} \partial_d F_{bc} \partial_i \partial_a \Phi + \dots + \chi_{11} F^{ab} \Omega^{cdi} \partial_d F_{ab} \partial_i \partial_c \Phi \right) \quad (27)$$

where ζ_i and χ_i are 20 arbitrary coefficients and Φ stands for the dilaton. Furthermore, the couplings of one graviton and three scalar fields in addition to the pull-back of couplings

in the (5), which was mentioned in the introduction, can appear only through the second fundamental form as follow:

$$S_{h\phi\phi\phi} = -\frac{\pi^2\alpha'^2 T_p}{48} \int d^{p+1}x \left(\lambda_1 \Omega_a^i \Omega_b^j \Omega_c^k \Omega_d^l \Omega_{cdj} \Omega_{cdi}^l + \lambda_2 \Omega_a^i \Omega_b^j \Omega_{cdj} \Omega_{cdi}^l + \lambda_3 \Omega_a^c \Omega_b^{abi} \Omega_{cdj} \Omega_{cdj}^l + \dots + \lambda_8 \Omega_{abi} \Omega^{abi} \Omega_{cdj} \Omega_{cdj}^l \right) \quad (28)$$

in which λ_i are 8 unknown coefficients that should be determined by imposing appropriate constraints. To assure the T-duality of one graviton and three scalar fields, we consider one dilaton and three scalar field couplings as follow:

$$S_{\Phi\phi\phi\phi} = -\frac{\pi^2\alpha'^2 T_p}{48} \int d^{p+1}x e^{-\Phi} \sqrt{-\tilde{G}} \left(\rho_1 \Omega_a^i \Omega_b^j \Omega_c^k \partial_i \Phi + \rho_2 \Omega_a^j \Omega_{bcj} \Omega^{bc} \partial_i \Phi + \rho_3 \Omega_a^{cj} \Omega^{ab} \Omega_{bcj} \partial_i \Phi + \rho_4 \Omega_a^i \Omega_{bcj} \Omega^{bcj} \partial_i \Phi \right) \quad (29)$$

Also, the couplings of one B-field, one gauge field and two scalar fields appear in two structures *i.e.*, $H\partial F\Omega\Omega$ and $\partial HF\Omega\Omega$. We consider these couplings as follow:

$$S_{B\phi\phi f}^{(1)} = -\frac{\pi^2\alpha'^2 T_p}{48} \int d^{p+1}x e^{-\Phi} \sqrt{-\tilde{G}} \left(\iota_1 H_{cde} \Omega^{bai} \Omega_c^a \partial_b F^{de} + \iota_2 H_{dij} \Omega^{bai} \Omega_a^c \partial_c F_b^d + \iota_3 H_{cde} \Omega_a^{ai} \Omega^{cb} \partial_b F^{de} + \dots + \iota_{14} H_{cde} \Omega_{bai} \Omega^{bai} \partial^e F^{cd} \right) \\ S_{B\phi\phi f}^{(2)} = -\frac{\pi^2\alpha'^2 T_p}{48} \int d^{p+1}x e^{-\Phi} \sqrt{-\tilde{G}} \left(\psi_1 F^{ab} \Omega_{cb}^j \Omega_a^i \partial_d H_{ij}^d - \psi_2 F^{ab} \Omega_a^i \Omega_d^{dj} \partial_b H_{cij} + \psi_3 F^{ab} \Omega_a^i \Omega_d^{dj} \partial_c H_{bij} + \dots + \psi_{25} F^{ab} \Omega_a^i \Omega_b^j \partial_j H_{cdi} \right) \quad (30)$$

where ι_i and α_i are 39 arbitrary coefficients that should be determined by imposing appropriate constraints. As explained before, some of the couplings of one B-field, one gauge field and two scalar fields have been included in Eq. (25). Finally, the couplings of one B-field and three gauge fields can be introduced:

$$S_{Bfff}^{(1)} = -\frac{\pi^2\alpha'^2 T_p}{48} \int d^{p+1}x e^{-\Phi} \sqrt{-\tilde{G}} \left(\tau_1 H_{def} \partial_a F^{ab} \partial_b F^{cd} \partial_c F^{ef} + \tau_2 H_{def} \partial_a F_c^d \partial_b F^{ef} \partial^c F^{ab} + \tau_3 H_{def} \partial_b F^{ef} \partial_c F_a^d \partial^c F^{ab} + \dots + \tau_{29} H_{def} \partial_c F_{ab} \partial^c F^{ab} \partial^f F^{de} \right) \\ S_{Bfff}^{(2)} = -\frac{\pi^2\alpha'^2 T_p}{48} \int d^{p+1}x e^{-\Phi} \sqrt{-\tilde{G}} \left(\gamma_1 \partial_b H_{def} F^{ab} \partial_a F^{cd} \partial_c F^{ef} + \gamma_2 \partial_b H_{aef} F^{ab} \partial_c F^{cd} \partial_d F^{ef} + \gamma_3 \partial_d H_{bef} F^{ab} \partial_a F^{cd} \partial_c F^{ef} + \dots + \gamma_{81} \partial_f H_{cde} F^{ab} \partial^c F_{ab} \partial^f F^{de} \right) \\ S_{Bfff}^{(3)} = -\frac{\pi^2\alpha'^2 T_p}{48} \int d^{p+1}x e^{-\Phi} \sqrt{-\tilde{G}} \left(\delta_1 \partial_c \mathcal{F}^{ef} \partial^c \mathcal{F}^{ab} \partial_d \mathcal{F}_{ef} \partial^d \mathcal{F}_{ab} + \delta_2 \partial_a \mathcal{F}^{ab} \partial_c \mathcal{F}^{ef} \partial_d \mathcal{F}_{ef} \partial^d \mathcal{F}_b^c + \delta_3 \partial_b \mathcal{F}_{ef} \partial^c \mathcal{F}^{ab} \partial_d \mathcal{F}_c^f \partial^e \mathcal{F}_a^d + \dots + \delta_{33} \partial_c \mathcal{F}_{ab} \partial^c \mathcal{F}^{ab} \partial_f \mathcal{F}_{de} \partial^f \mathcal{F}^{de} \right) \quad (31)$$

in which τ_i , γ_i and δ_i are 143 arbitrary coefficients that should be deduced by imposing linear dualities.

Now we consider the sum of the couplings in (5), (24), (25), (26), (27), (28), (29), (30), (31) *i.e.*,

$$S_p = S_{h\phi ff}^{(1)} + S_{h\phi ff}^{(2)} + S_{\Phi\phi ff}^{(1)} + S_{\Phi\phi ff}^{(2)} + S_{h\phi\phi\phi} + S_{\Phi\phi\phi\phi} \\ + S_{B\phi\phi f}^{(1)} + S_{B\phi\phi f}^{(2)} + S_{B\phi ff}^{(1)} + S_{B\phi ff}^{(2)} + S_{B\phi ff}^{(3)} + S_{p,l} \quad (32)$$

as the effective action, in α'^2 order of one closed and three open strings, and apply the T-duality constraint (14). This procedure gives the following relations between the constants:

$$\begin{aligned} \iota_4 = 0, \quad \iota_{18} = -1/2, \quad \eta_{37} = 1 + \eta_{13} + \eta_{17} - \eta_{21} - \eta_{22} - \eta_{28} - \eta_{29} - \eta_{36} + 2\eta_{19} \\ \gamma_{33} = 1 - \gamma_{13} - \gamma_{14} - \gamma_{32}, \quad \gamma_{35} = -1 + \gamma_{13} + \gamma_{14} + \gamma_{28} - \gamma_{29} + \gamma_{30} + \gamma_{32} + 2\gamma_{34} \\ \gamma_{65} = -1 + \gamma_{28} - \gamma_{29} + \gamma_{62} - \gamma_{63} + \gamma_{64}, \quad \gamma_7 = -3/4 - \gamma_{11}/2 + \gamma_{14}/2 \\ -\gamma_{22} + \gamma_{23}/2 - \gamma_{25}/2 + \gamma_{26}/2 - \gamma_{27}/2 - \gamma_{29}/2 - \gamma_3 - \gamma_4 + \gamma_{40}/2 + \gamma_{41} + \gamma_{42}/2 \\ + \gamma_{44} + \gamma_{47}/2 - \gamma_{49}/2 + \gamma_{59}/2 - \gamma_6 + \gamma_{60}/2 + \gamma_{62}/2 - \gamma_{63}/2 + \gamma_{67}/2 + \gamma_{68}/2 \\ \eta_{39} = -2 - \eta_{13}, \quad \eta_5 = 1 - 4\gamma_1 - 4\gamma_{10} + 2\gamma_{11} + 2\gamma_{12} - 2\gamma_{13} - 4\gamma_{14} - 2\gamma_{26} + 2\gamma_{27} \\ -2\gamma_{28} + 2\gamma_{29} + 4\gamma_4 + 2\gamma_{40} - 2\gamma_{42} + 4\gamma_{43} - 4\gamma_{44} - 2\gamma_{47} + 2\gamma_{49} + 2\gamma_{58} - 2\gamma_{60} - 2\gamma_{62} \\ + 2\gamma_{63} - \eta_{17} + \eta_{26}, \quad \iota_8 = -1/4 - \iota_{12}/2\gamma_{17} = 1/2 - \gamma_{15} - \gamma_{16}, \quad \gamma_{21} = \gamma_{19}/2 - \gamma_{20}/2 \\ \gamma_{35} = -1 + \gamma_{13} + \gamma_{14} - \gamma_{28} - \gamma_{29} + \gamma_{30} + \gamma_{32} + 2\gamma_{34}, \quad \dots \end{aligned} \quad (33)$$

where the dots, here and in the following, refers to the constraints which do not include any constant number and only provide a relationship between unknown coefficients.

As can be seen, all coefficients of the effective action, *i.e.*, S_p , in the (32) are not fixed by imposing consistency of the couplings with the linear T-duality. Thus we need further limitations to constrain the effective action. These constraints might be chosen as the imposition of the consistency with S-duality. For example, by imposing S-duality, it is possible to see that one dilaton and three scalar fields in (29) must be vanished in the world volume of D₃-brane in the Einstein frame. This produces the following constraint:

$$\delta_{25} = 1/2 - \delta_{16}/2 - \delta_{17}/2 - \delta_{18} - \delta_{19}/2 - \delta_{23} \quad (34)$$

In addition, S-duality constrains the couplings of one graviton, one scalar field and two gauge fields in (24) and (25). These couplings in the world volume of D₃-brane action must appear in the S-duality invariant structures $R\Omega\partial\mathcal{F}^T\mathcal{M}_0\mathcal{F} = e^{-\phi_0}R\Omega(\partial(*F)(*F) + \partial FF)$. This condition provides the following constraints:

$$\begin{aligned} \delta_8 &= -2 + 4\delta_1 - \delta_{15} + \delta_{18} + \delta_{19} - 2\delta_{21} + 2\delta_{25} - \delta_{27} - \delta_7, \\ \gamma_{63} &= -1 + 2\gamma_1 + 2\gamma_{10} - \gamma_{11} - \gamma_{12} + \gamma_{13} + 2\gamma_{14} + \gamma_{26} - \gamma_{27} + \gamma_{28} - \gamma_{29} - 2\gamma_4 - \gamma_{40} \\ &+ \gamma_{42} - 2\gamma_{43} + 2\gamma_{44} + \gamma_{47} - \gamma_{58} + \gamma_{60} + \gamma_{62}, \quad \dots \end{aligned} \quad (35)$$

By applying T-duality constraint in (33) and S-duality constraint in (34) and (35) to the one dilaton, one scalar and two gauge fields couplings in (26) and (27) we found that these couplings satisfy the S-dual structure, *i.e.*, the second equation in (21), without any constraint in the coefficient.

As the final S-duality constrain, we check the consistency of the B-fields couplings with linear S-duality according to (20), *i.e.*, the S-dual multiplet, and find the following constraint:

$$\begin{aligned}\delta_8 &= -2 + 4\delta_1 - \delta_{15} + \delta_{18} + \delta_{19} - 2\delta_{21} + 2\delta_{25} - \delta_{27} - \delta_7 \\ \gamma_{63} &= 7/4 + 2\gamma_1 + 2\gamma_{10} - \gamma_{11} - \gamma_{12} + 2\gamma_{13} + 3\gamma_{14} + \gamma_{26} - \gamma_{27} + 2\gamma_{28} \\ &- 4\gamma_{29} - 2\gamma_4 - 2\gamma_{42} - 4\gamma_{43} + 2\gamma_{44} + 4\gamma_{47} - \gamma_{49} - \gamma_{58} + \gamma_{60} + \gamma_{62}, \dots\end{aligned}$$

Now we impose all constraints in (33), (34) and (35), resulted from T-duality and S-duality, on the DBI effective action (32) and obtain the T-dual and S-dual invariance effective action.

On the other hand the Riemann curvature satisfies the cyclic symmetry while the field strengths satisfy the Bianchi identities. Therefore, these symmetries have to be imposed in S_p^{DBI} . To perform this step, we write all field strengths in terms of their corresponding potentials and write the Riemann curvature in terms of

$$R_{abcd} = \partial_b \partial_c h_{ad} + \partial_a \partial_d h_{bc} - \partial_b \partial_d h_{ac} - \partial_a \partial_c h_{bd} \quad (36)$$

Then we find that the coefficients γ_i with $i = 1, 2, 3, 4, 5, 6, 16, 18, 20, 24, 26, 27, 30, 32, 34, 36, 37, 45, 46, 53, 54, 55, 56, 58, 59, 60, 61, 62, 64, 66, 67, 68, 83, 74, 78, 76, 77, 78, 80$ and η_i with $i = 1, 2, 3, 7, 11, 12, 19, 24, 26, 28, 29, 30, 31, 33, 36, 41, 44, 47, 50, 53, 55, 56$ and δ_i with $i = 2, 3, 7, 10, 12, 13, 14, 19, 20, 22, 24, 30, 31$ and ζ_i with $i = 1, 5, 8$ and τ_i with $i = 1, 2, 4, 5, 10, 11, 12, 13, 14, 15, 16, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29$ and ϕ_i with $i = 1, 13, 16, 21, 22$ and χ_i with $i = 5, 7, 10$ and ψ_i with $i = 10, 11, 12, 15, 16$ disappear from the action. Consequently, the couplings with the above coefficients display only the cyclic symmetry and the Bianchi identities. Thus, we ignore such terms in the DBI effective action. Finally, we find that the couplings with coefficients γ_i with $i = 10, 11, 12, 13, 14, 15, 19, 22, 23, 25, 28, 29, 38, 39, 40, 41, 42, 43, 44, 47, 48, 49$ and η_i with $i = 13, 17, 21, 22$ and ζ_i with $i = 2, 3$ and ι_i with $i = 1, 11, 12$ are total derivative terms, so they can be eliminated from the DBI effective action too.

By Imposing T-duality and S-duality constraints plus Bianchi identity and cyclic symmetry while neglecting the total derivative couplings from the effective action, we find that all unknown coefficients in (32) will be fixed except δ_i with $i = 1, 6, 15, 16, 17, 18, 21, 23, 27, 29, 32$.

One may fix the above coefficients by comparing the couplings $S_{h\phi\phi\phi}$ in (28), $S_{Bff}^{(3)}$ in (31) and $S_{h\phi ff}^{(2)}$ in (25) with the couplings in (7), (8) and (9) respectively after applying all mentioned constraints. By comparing (28) with (7) we find the following relations:

$$\delta_{15} = 1, \quad \delta_{29} = -2(2 + \delta_{32} - \delta_6)$$

while comparing (31) with (8) yields the the following constraints:

$$\delta_1 = \delta_6 = \delta_{16} = \delta_{17} = \delta_{18} = \delta_{21} = 0, \quad \delta_{23} = 2, \quad \delta_{32} = -2$$

and comparing (25) with (9) provides the following constraints:

$$\delta_{27} = -4$$

To conclude, we point out that by imposing all above constraints to the effective action of (32), we could find all arbitrary coefficients and add them other couplings of one NSNS closed string and three NS open string correction to the DBI action at the α'^2 order. As the result, the dilaton couplings can be found as follow:

$$S_{\Phi\phi\phi\phi} = -\frac{\pi^2\alpha'^2 T_p}{48} \int d^{p+1}x \sqrt{-\tilde{G}_{ab}} \left[F^{ab}\Omega_d{}^d{}_i \partial_c F_{ab} \partial^i \partial^c \Phi + F^{ab}\Omega_d{}^d{}_{ai} \partial_c F_{bd} \partial^i \partial^c \Phi \right. \quad (37)$$

$$\begin{aligned} & -F^{ab}\Omega_d{}^d{}_{ci} \partial_d F_{ab} \partial^i \partial^c \Phi + F^{ab}\Omega_d{}^d{}_{ai} \partial_d F_{bc} \partial^i \partial^c \Phi - 2F^{ab}\Omega_{cai} \partial_d F_b{}^d \partial^i \partial^c \Phi \\ & + \frac{1}{2}\Omega_a{}^a{}_i \Omega_b{}^{bj} \Omega_d{}^d{}_j \partial^i \Phi - 5\Omega_{ba}{}^j \Omega^{ba}{}_i \Omega_d{}^d{}_j \partial^i \Phi + \Omega^{ba}{}_i \Omega_{dbj} \Omega^d{}_a{}^j \partial^i \Phi \\ & \left. + \frac{15}{2}\Omega_a{}^a{}_i \Omega_{dbj} \Omega^{dbj} \partial^i \Phi + 4\Omega^{ba}{}_i \partial_b F_a{}^c \partial_d F_c{}^d \partial^i \Phi + 4\Omega_a{}^a{}_i \partial_d F_{bc} \partial^d F^{bc} \partial^i \Phi \right] \quad (38) \end{aligned}$$

while the other couplings are reported formerly in (7), (8), (9), (10), (11) and (12).

On the other hand, the D-brane effective action at the order α'^2 should be consistent with S-matrix elements. It would be interesting to verify above couplings to be consistent with the S-matrix elements of one NSNS and three NS states.

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